Transmission Coefficients over Humps and Indentations

Hui Xu, Spencer Sherwin, Philip Hall, Xuesong Wu

1. Motivation

The behaviour of Tollmien-Schlichting (T-S) waves experiencing small-localised distortions within the boundary layer along a flat plate has both practical and theoretical significances on natural transition. Practically, understanding effects of humps and indentations on the growth of TS waves has a significant impact on the mechanism of triggering laminar-transition process. Linear analysis based on the triple-deck asymptotic method predicts that humps and indentations have opposite effect on attenuating and amplifying the TS waves. Understanding the real behaviour can benefit from numerical investigation, which is also a way to validate the theoretical finding. Numerically, both linear and non-linear regimes of humps and indentations should be explored to observe the behaviour of the TS waves. Another issue is that quantifying the influences of humps and indentations on the TS waves offers a realization such that laminar-turbulent transition prediction strategy can be improved or some possible promotion of laminar-turbulent transition can be well-understood. The quantification of the behaviour of the TS wave undergoing hump/indentation distortions is formulated by transmission coefficient.

2. Research

The research is focused on both numerical and theoretical aspects. Numerically, having access to linear and non-linear regimes needs to resolve the geometry of wall imperfection exactly and solve the related equations with high precision. With high-order curved elements generated around the wall imprecations by an analytical mapping, spectral element method is used to capture the physics of the TS waves in the linear and nonlinear regimes of humps and indentations. The asymptotical prediction is well understood based on the analyses of linear and non-linear results. The values of the exponential parameters of transmission coefficient emerge differently in the linear and non-linear regimes of humps and indentations. Whilst, the results indicate that the linear approximation requires a stringent condition, which is due to degeneracy of the linear mechanism. So, generally, the same amplification effects can be found for both humps and indentation but the transmission coefficient is very sensitive to non-linearity.

3. Application

The behaviour of hump/indentation distortions on the TS waves clarifies ambiguous understanding on laminar-turbulent transition from numerical and theoretical points of view. The key findings are:

- Linear-term is not dominant in the transmission coefficient expression;
- Quadratic-term effect makes humps and indentations have the same effect;
- Without separation, quadratic-term dominates the TS waves' behaviour.

The founding indicates there exists a quasi universal scaling in the transmission coefficient expression which can be leverage to facilitate the use of transition coefficient in industry and address the transition promotion induced by a single isolated roughness or multiple isolated roughness elements.